

Injection-locked diode laser current modulation for Pound-Drever-Hall frequency stabilization using transfer cavities

C. E. Liekhus-Schmaltz,¹ R. Mantifel,^{1,2} M. Torabifard,¹ I. B. Burgess,^{1,3} J. D. D. Martin¹

¹ *Department of Physics and Astronomy and Institute for Quantum Computing,
University of Waterloo, Waterloo ON, N2L 3G1, Canada*

² *Currently with Department of Physics, McGill University,
3600 rue University, Montréal QC, H3A 2T8, Canada and*

³ *Currently with School of Engineering and Applied Sciences,
Harvard University, Cambridge, MA, 02138, USA*

(Dated: December 30, 2011)

A phase modulated RF current source is applied to an injection locked diode laser operating at 780 nm. This produces tunable phase modulated sidebands of the laser suitable for stabilizing the length of an optical transfer cavity using the Pound-Drever-Hall technique. The Pound-Drever-Hall signal is anti-symmetric about the lock point, despite the presence of significant diode laser amplitude modulation. The stabilized optical transfer cavity is used to frequency stabilize a 776 nm external cavity diode laser. The stability and tunability of this transfer cavity locked laser is established by observation of the hyperfine components of the $^{87}\text{Rb } 5P_{3/2} - 5D_{5/2}$ transition in a vapor cell.

Lasers are often frequency stabilized using optical cavity modes. Although stable passive cavities can be constructed [1], it is also possible to *actively* stabilize a cavity, using a second laser that is frequency stabilized using an atomic reference [2]. The cavity length can be adjusted with a high bandwidth piezoelectric actuator (PZT) [3] to keep it in resonance with the reference laser. Since the cavity *transfers* the stability of a reference laser to a target laser it is often referred to as a “transfer cavity.” The two lasers may be at very different wavelengths, provided the cavity finesse remains sufficiently high at the two wavelengths.

To stabilize the target laser at an arbitrary frequency, a tunable frequency shift of either the target or reference laser is necessary. This shift may be obtained by electro-optic modulators [2], acousto-optic modulators [4], or current-modulated injection-locked diode lasers [5].

We demonstrate that a tunable frequency shift and the modulation required for Pound-Drever-Hall (PDH) locking [6] can be obtained by applying a phase modulated RF current to an injection locked slave diode laser. This modulation produces tunable phase modulated sidebands of the reference laser. By locking a transfer cavity to one of these sidebands, its length is controlled by a stable, tunable reference frequency, and the frequency of a target laser may be precisely controlled.

The mechanism for producing sidebands can be intuitively understood from the general behavior that injection locked systems exhibit [7]: the phase difference between the injected and oscillator signal is $\phi = \sin^{-1}(\{\omega_0 - \omega_1\}/\omega_m)$, where ω_0 is the free running oscillator angular frequency, ω_1 is the injected signal angular frequency, and ω_m is the locking half-width (the total locking range being $2\omega_m$). As the phase difference between the injected and locked oscillator signal depends on the detuning between them, modulation of the locked oscillator resonance frequency will phase modulate its output.

The lasing frequency of a conventional Fabry-Perot diode laser can be rapidly changed with current. Kobayashi and Kimura [8] demonstrated that by sinusoidally modulating the current applied to an injection locked diode laser, they could observe frequency sidebands corresponding to phase modulation. In particular, provided the frequency content of the injection current $I(t) = I_{\text{dc}} + \Delta I(t)$ is within the locking half-width ω_m , the output of the slave laser is of the form:

$$\tilde{E} = \tilde{E}_0 \exp \{j(\omega_0 t + k\Delta I(t))\}, \quad (1)$$

where $\Delta I(t)$ represents the deviation of the current from the dc value corresponding to $\omega_0 = \omega_1$, and $k \approx (\partial\omega_0/\partial I)/\omega_m$. Sinusoidal modulation of $\Delta I(t)$ produces phase modulation: $\tilde{E} = \tilde{E}_0 \exp \{j(\omega_0 t + \alpha \sin[\delta t])\}$, with sidebands in frequency space at $\omega_0 \pm \delta$ with powers of $P_1 = P[J_1(\alpha)]^2$ where P is the total power. As $\delta \rightarrow \omega_m$ this ceases to be true and the output is more accurately described as being frequency-modulated rather than phase-modulated [8]. This simple picture neglects the amplitude modulation that must accompany diode laser current modulation. We will return to this important issue after presentation of the experimental results, which unexpectedly show no adverse effects due to residual amplitude modulation.

Here we consider a current source with phase modulation at an angular frequency Ω that is expected to produce a slave laser output:

$$\tilde{E} = \tilde{E}_0 \exp \{j(\omega_0 t + \alpha \sin[\delta t + \beta \sin(\Omega t)])\}. \quad (2)$$

With $\delta \gg \Omega$ this output corresponds to tunable phase modulated sidebands centered around $\omega_0 \pm \delta$. The cavity may be locked to one of the sidebands using the standard PDH scheme where the modulation frequency is Ω . The cavity length may then be scanned by varying δ (keeping Ω constant). If a second “target laser” is locked to

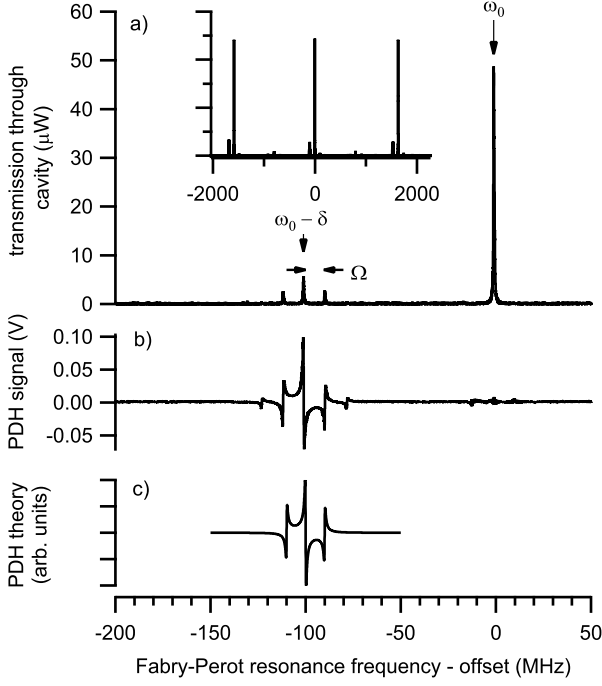


FIG. 2. (a) Fabry-Perot transmission spectra, and (b) PDH error signal for $\delta/(2\pi) = 100$ MHz observed using a scanning Fabry-Perot cavity with a free spectral range (FSR) of 1.6 GHz and finesse (\mathcal{F}) of 2500. No offset has been added to this signal. (c) Calculated PDH error signal shown for comparison (see, for example [11]).

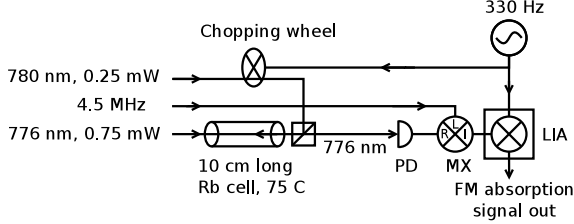


FIG. 3. Scheme to monitor 776 nm ECDL tunability and frequency stability using the ^{87}Rb , $5P_{3/2} - 5D_{5/2}$ transitions. LIA: lock-in amplifier MX: mixer, PD: photodiode

modulation.

In view of the large difference between the magnitudes of the $\omega_0 \pm \delta$ sidebands (the $\omega_0 + \delta$ sideband is barely visible in the inset to Fig. 2) it is perhaps surprising that the $\omega_0 - \delta + \Omega$ and $\omega_0 - \delta - \Omega$ sidebands have the same magnitude. It is often observed that direct current modulation of a diode laser with a *single tone* at Ω produces asymmetric sidebands, and thus imperfect PDH signals. However symmetric *secondary* sideband magnitudes are consistent with a straightforward model of the modulated slave laser output that includes amplitude modulation.

To rationalize why the secondary sideband magnitudes are equal, consider a model for the output of the slave

laser that includes amplitude modulation:

$$\frac{\tilde{E}}{\tilde{E}_0} = (1 + A \sin[\phi] + B \cos[\phi]) \exp\{j(\omega_0 t + \alpha \sin[\phi])\}, \quad (3)$$

where $\phi = \delta t + \beta \sin(\Omega t)$. For relatively low δ we might expect that the phase and amplitude modulation would be in phase and $B = 0$. However, this is inconsistent with our observation of asymmetric sideband amplitudes at $\omega_0 + \delta$ and $\omega_0 - \delta$ when we apply a single tone at δ (see, for example, Table II of [13]). There must be simultaneous amplitude and phase modulation, and a phase shift between them. This phase shift may be calculated from a detailed knowledge of the laser and the operating parameters, as Lidoyne *et al.* [14] have shown.

Using the Jacobi-Anger identity to expand Eq. (3) we find that the amplitude of the $\omega_0 - \delta + \Omega$ frequency component is

$$\begin{aligned} \frac{\tilde{E}_{\omega_0 - \delta + \Omega}}{\tilde{E}_0} = & J_1(\alpha)J_1(\beta) - \frac{(Aj + B)}{2}J_1(\beta)J_0(\alpha) \\ & + \frac{(-Aj + B)}{2}J_2(\alpha) \sum_{\ell=-\infty}^{\infty} J_{\ell}(\beta)J_{\ell-1}(2\beta), \end{aligned} \quad (4)$$

where J_{ℓ} is a Bessel function of order ℓ . The $\omega_0 - \delta - \Omega$ frequency component has the same magnitude but is of opposite sign. Thus these sidebands are suitable for the PDH technique [6, 11], which is consistent with our observations. A similar result is found for the $\omega_0 + \delta + \Omega$ and $\omega_0 + \delta - \Omega$ frequency components.

To demonstrate the tunability of a transfer cavity locked using the modulated injection locked slave laser, we have stabilized the frequency of a 776 nm external cavity diode laser. The transfer cavity had a $FSR = 1.9$ GHz and $\mathcal{F} = 350$. Its length is locked using a PM tunable sideband of the slave laser, as shown in Fig. 1. The 776 nm diode laser is then locked to a cavity resonance using PDH locking. This laser is current modulated at 4.5 MHz to provide PM sidebands suitable for locking.

The ^{87}Rb $5P_{3/2} - 5D_{5/2}$ transition [15] is used to observe the locked 776 nm laser's tunability and frequency stability. A 780 nm counter-propagating beam is sent through a Rb vapor cell exciting the $5P_{3/2}$, $F' = 3$ states, and the 776 nm absorption measured with a photodiode (Fig. 3). We demodulate the absorption using the 4.5 MHz modulating source to produce an FM, dispersion-like absorption signal [9]. To improve signal to noise, the 780 nm beam intensity is modulated with a chopping wheel, and both the absorption and FM signals detected with a lock-in amplifier. By varying δ we may scan the frequency of the 776 nm laser over the absorption lines (see Fig. 4a).

As with Grove *et al.* [15] we can compare the observed frequency differences between absorption lines with the results of Nez *et al.* [16]. The correspondence

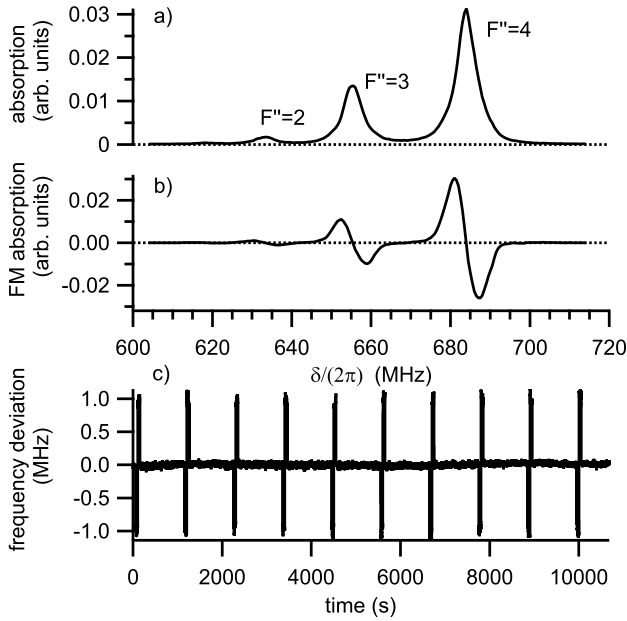


FIG. 4. (a) Absorption spectrum of the $5P_{3/2} - 5D_{5/2}$ transition of ^{87}Rb (b) FM absorption spectrum (see Fig. 3) (c) Frequency stability of the locked 776 nm laser as a function of time, monitored using the FM absorption signal (0.3 s time constant). The RF frequency is set to place the 776 nm laser at the $5P_{3/2}$, $F' = 3 \rightarrow 5D_{5/2}$, $F'' = 4$ transition and the FM signal is used as a frequency discriminator. Periodically stepping $\delta/(2\pi)$ up and down by 1 MHz allows conversion of the FM signal to frequency deviation.

between a frequency change in the locked laser Δf_{776} and a frequency change in the tunable sideband Δf_{780} is: $\Delta f_{776}/\Delta f_{780} \approx f_{776}/f_{780} \approx 1.0055$. Using this factor, we determine from the absorption spectrum in Fig. 4a) that the energy level difference between $5D_{5/2}$ $F'' = 4$ and $F'' = 3$ is 28.82 ± 0.03 MHz, and between $F'' = 3$ and $F'' = 2$ it is 22.7 ± 0.3 MHz. These results are consistent with 28.82 ± 0.01 MHz and 22.96 ± 0.01 MHz from [16].

As δ is varied the dc offset of the PDH signal varies, possibly due to amplitude modulation of the slave laser at 10 MHz. Although the magnitude of this offset is typically small compared to the peak PDH signal, this varying offset can introduce non-linearities in the scanned laser frequency. In general this effect is reduced for larger δ . For example, by recording PDH spectra as a function of δ for the cavity of Fig. 2 we have established an upper bound of 50 kHz on scan non-linearities, and a variation in the PDH signal magnitude of less than 50% for $\delta = 500$ MHz to $\delta = 750$ MHz. Although we have not done so, the influence of the dc offset can be reduced by deriving the PDH signal from the difference of two photodiode signals: one which measures the light incident on the cavity, and the other which measures the light reflected from the cavity. The relative powers incident

on each photodiode should be adjusted to null the PDH error signal when the cavity is not resonant with the incident light.

The long-term frequency fluctuations of the 776 nm frequency stabilized laser may be monitored using the $5P_{3/2} - 5D_{5/2}$ FM signal. By setting δ to the strongest absorption line ($F'' = 4$), the FM signal (Fig. 4b) may be used to provide an upper bound on the locked laser frequency fluctuations (Fig. 4c). The observed frequency stability is sufficient for many laser locking applications. For example, this system has been in regular use for several months, to frequency stabilize a Ti:sapphire laser operating at 960 nm for the excitation of laser cooled atoms to Rydberg states [17]. The lock is typically maintained over several hours, usually limited by the need to relock the master laser.

In summary, we have demonstrated that with the appropriate current modulation, tunable phase modulated sidebands can be put on injection locked diode lasers. These sidebands may be used to stabilize and vary the lengths of PZT based optical transfer cavities using the PDH technique.

We thank L. Jones for providing the heated Rb cell. This work was supported by NSERC.

-
- [1] A. D. Ludlow, X. Huang, M. Notcutt, T. Zanon-Willette, S. M. Foreman, M. M. Boyd, S. Blatt, and J. Ye, "Compact, thermal-noise-limited optical cavity for diode laser stabilization at 1×10^{-15} ," *Opt. Lett.* **32**, 641–643 (2007).
 - [2] B. Burghardt, W. Jitschin, and G. Meisel, "Precise rf tuning for cw dye lasers," *Appl. Phys.* **20**, 141–146 (1979).
 - [3] T. C. Briles, D. C. Yost, A. Cingöz, J. Ye, and T. R. Schibli, "Simple piezoelectric-actuated mirror with 180 kHz servo bandwidth," *Opt. Express* **18**, 9739–9746 (2010).
 - [4] D. F. Plusquellic, O. Votava, and D. J. Nesbitt, "Absolute frequency stabilization of an injection-seeded optical parametric oscillator," *Appl. Opt.* **35**, 1464–1472 (1996).
 - [5] P. Bohlouli-Zanjani, K. Afrousheh, and J. D. D. Martin, "Optical transfer cavity stabilization using current-modulated injection-locked diode lasers," *Rev. Sci. Instrum.* **77**, 093105 (2006).
 - [6] R. W. P. Drever, J. L. Hall, F. V. Kowalski, J. Hough, G. M. Ford, A. J. Munley, and H. Ward, "Laser phase and frequency stabilization using an optical resonator," *Appl. Phys. B* **31**, 97–105 (1983).
 - [7] A. Siegman, *Lasers* (University Science Books, 1986).
 - [8] S. Kobayashi and T. Kimura, "Optical phase modulation in an injection locked AlGaAs semiconductor laser," *IEEE J. Quantum Electron.* **18**, 1662 – 1669 (1982).
 - [9] G. C. Bjorklund, "Frequency-modulation spectroscopy: a new method for measuring weak absorptions and dispersions," *Opt. Lett.* **5**, 15–17 (1980).
 - [10] R. V. Pound, "Electronic frequency stabilization of microwave oscillators," *Rev. Sci. Instrum.* **17**, 490 (1946).
 - [11] C. E. Liekhuis-Schmaltz and J. D. D. Martin, "Understanding Pound-Drever-Hall locking using voltage con-

- trolled radio-frequency oscillators: An undergraduate experiment,” ArXiv:1108.0960.
- [12] S. H. Youn, M. Lu, U. Ray, and B. L. Lev, “Dysprosium magneto-optical traps,” *Phys. Rev. A* **82**, 043425 (2010).
 - [13] S. Kobayashi, Y. Yamamoto, M. Ito, and T. Kimura, “Direct frequency modulation in AlGaAs semiconductor lasers,” *IEEE J. Quantum Electron.* **18**, 582–595 (1982).
 - [14] O. Lidoyne, P. Gallion, and D. Erasme, “Modulation properties of an injection-locked semiconductor laser,” *IEEE J. Quantum Electron.* **27**, 344–351 (1991).
 - [15] T. T. Grove, V. Sanchez-Villicana, B. C. Duncan, S. Maleki, and P. L. Gould, “Two-photon two-color diode laser spectroscopy of the Rb $5D_{5/2}$ state,” *Physica Scripta* **52**, 271 (1995).
 - [16] F. Nez, F. Biraben, R. Felder, and Y. Millerioux, “Optical frequency determination of the hyperfine components of the $5S_{1/2}$ - $5D_{3/2}$ two-photon transitions in rubidium,” *Opt. Commun.* **102**, 432–438 (1993).
 - [17] J. A. Petrus, P. Bohlouli-Zanjani, and J. D. D. Martin, “ac electric-field-induced resonant energy transfer between cold Rydberg atoms,” *J. Phys. B: At. Mol. Opt. Phys.* **41**, 245001 (2008).